10) <u>Causality Check of LTI Systems Using the Impulse Response</u>

- Recall: A LTI system is said to be causal if the output $y(n)$

- Goal: Develop the condition for an LTI system to be causal in terms of $h(n)$

- Proof: Define 2 sequences $x_1(n)$ and $x_2(n)$ so that $x_1(n) = x_2(n) \quad n \; n_0$

  – system outputs:

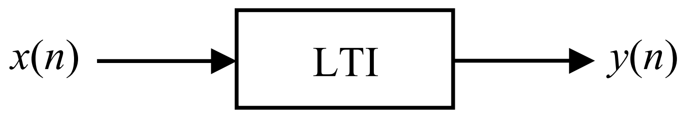
$$y_1(n) =$$

$$=$$

$$y_2(n) =$$

$$=$$

11) LTI Systems and Linear Difference Equations



- An important subclass of LTI systems is characterized by a linear difference equation of the form:

$$y(n) =$$

(1)

- Input/output relationship may be represented by a block diagram

- Example:

  $$y(n)=b_0x(n)+b_1x(n-1) + b_2x(n-2)$$

  $$y(n)=a_1y(n-1)+a_2y(n-2)+b_0x(n)+b_1x(n-1) + b_2x(n-2)$$

12) Impulse & Step Response and Difference Equation

- Recall that

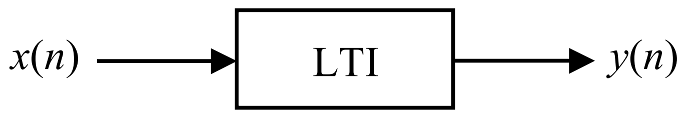
Observation: The only thing we need to know about the system to compute the output is h(n)

Example: You are given the I/O relationship to a system

$$y(n) = 0.6\,y(n-1) + 2\,x(n)$$

- Compute and plot the impulse response h(n)
- Compute and plot the unit step response

13) <u>LTI Systems and Initial Condition Response</u>



• Definition: The **initial condition (IC) response** of a LTI system is the part of the response caused by initial conditions (sometimes called the “zero-input response”

• Why is the IC response important ?

$\Rightarrow$ it plays a role in evaluating the stability of the system

• Definition: The LTI system is said to be stable if the IC response approaches 0 as $n \to \infty$

- Example: $y(n)=ay(n-1)+x(n)$ with $y(0)=K$

- Recall:
  The LTI system is said to be stable if the IC response approaches 0 as $n \to \infty$

- Example: $y(n)=0.6y(n-1)+2x(n)$ with $y(0)=1$

- IC response and generic LTI IO relationships

$$y(n)-a_1 y(n-1)-\ldots-a_N y(n-N)=b_0 x(n)+\cdots+b_\ell x(n-\ell)$$

- Recall:

The LTI system is said to be stable if the IC response approaches 0 as $n \to \infty$

- Special case for the solution when the characteristic polynomial has multiple roots.

Given the IO relationship:

$$y(n)-a_1y(n-1)-\ldots-a_Ny(n-N)=b_0x(n)+\cdots+b_\ell x(n-\ell)$$

Resulting characteristic equation is of the form:

Assume the characteristic equation has a root r_1 of multiplicity L and the other roots are distinct

- Example: $y(n)=-0.3y(n-1)+0.4y(n-2)+0.5x(n)+0.5x(n-1)$
$y(-1)=y(-2)=1$

  Compute:

    1) the characteristic equation
    2) the characteristic roots
    3) the stability condition of the system

- How to check that the solution is valid ?

14) Linear Difference Equation and Total Solution

$$x(n) \longrightarrow \boxed{\text{LTI}} \longrightarrow y(n)$$

$$y(n) - a_1 y(n-1) - \ldots - a_N y(n-N) = b_0 x(n) + \cdots + b_\ell x(n-\ell)$$

- Total solution calculation
  - Procedure to compute the solution to (1) is similar to solving differential equations

$$\boxed{y(n) = y_c(n) + y_p(n)}$$

Solution of (1) when $x(n) = 0$	Solution of (1) for $x(n) \neq 0$
– complementary solution –	– particular solution –
Solution depends on the initial conditions	Solution depends on the input only

Constants computed by using the total solution

a) Complementary Solution

- Equation to solve for $n \geq 0$

$$y(n) - a_1 y(n-1) - \ldots - a_N y(n-N) = 0$$

initial condition $y(-1) \ldots y(-N)$ given

- First assume $y_c(n) = r^n$

$$\Rightarrow \quad y_c(n) - a_1 y_c(n-1) - \ldots - a_N y_c(n-N) =$$

$$=$$

$\Rightarrow$ General form of complementary solution is is given by:

$y_c(n)=$

b) Particular Solution

- Equation to solve

$$y(n)-a_1 y(n-1)-\ldots-a_N y(n-N)=b_0 x(n)+\cdots+b_\ell x(n-\ell)$$

- Pick $x(n)$ and assume that particular solution is of the same general form as the specified $x(n)$

Shape of x(n)	Shape of particular solution
C_1	C_2
Kn	$K_1 n+K_2$
$K_1 a^n$	$K_2 a^n$
$K a^n \cos(n\theta)$	$K_1 a^n \cos(n\theta+K_2)$
$K\cos(n\theta)$	$K_1 \cos(n\theta+K_2)$

Example:

Find the total solution for $n \geq 0$ for

$$y(n) + y(n-1) - 6y(n-2) = x(n)$$

a) When $x(n) = 8u(n)$ and initial conditions $y(-1) = 1$, $y(-2) = -1$

b) When $x(n) = 2^n u(n)$ with $y(-1) = 1$, $y(-2) = -1$

- **Example:** Find the solution for $n \geq 0$ to

$$y(n)+0.5y(n-1)+0.4y(n-2)=0$$

when initial conditions are $y(-1) = -0.8$; $y(-2) = 0.5$

Example:

Find the total solution for $n \geq 0$ for

$$y(n) - y(n-1) + 0.5y(n-2) = 0$$

initial conditions $y(-1) = -0.8$, $y(-2) = 0.5$

Example:

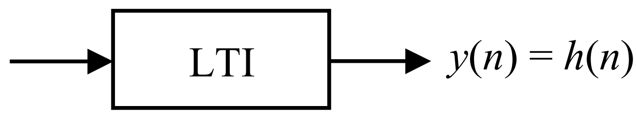
Find the total solution for $n \geq 0$ for

$$y(n) + y(n-2) = x(n) + x(n-1)$$

initial conditions $y(-2) = -10$, $y(-1) = 0$, $x(n) = 10u(n)$

15) Impulse Response and Difference Equation

- Recall that



Example: Compute the impulse response to the system

$$y(n)+y(n-1)-6y(n-2)=x(n)$$

with $x(n) = 8u(n)$; $y(-1) = 1$; $y(-2) = -1$

16) System Output Computation Using MATAB

- Given

$$y(n) + a_1 y(n-1) + \cdots + a_N y(n-N) = + b_0 x(n) + \cdots + b_L x(n-L)$$

- Pack coefficients in two vectors:

A = [

B = [

- Output vector $\underline{y}\{y(0), y(1), \cdots, y(k)\}$

to input vector $\underline{x}\{x(0), x(1), \cdots, x(k)\}$

is given by

$y =$

Example: Assume you want to average 10 input data points

$$y(n) = \frac{1}{10}\left(x(n) + \cdots + x(n-9)\right)$$

A =

B =